Hadronic Annihilation Decay Rates of P-wave Heavy Quarkonia with Both Relativistic and QCD Radiative Corrections

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Abstract

Hadronic annihilation decay rates of P-wave heavy quarkonia are given to next-to-leading order in both α_s and v^2. They include ten nonperturbative parameters, which can be rigorously defined as the matrix elements of color-singlet and color-octet operators in NRQCD. We expect these papameters will be determined from lattice calculations in future.

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BBL (Bodwin, Braaten and Lepage) [1] factorization formalism has provided a rigorous QCD prediction for heavy quarkonia decay and production. In this formalism, the decay rate can be written in terms of two expansions in strong coupling constant α_s and v^2, the relative velocity of quark and antiquark in quarkonium. Nonrelativistic results including QCD radiactive corrections have been given [2–4] for S-wave and P-wave decay rates. However, in heavy quarkonium system, especially for charmonium, relativistic effects are very important. In $c\bar{c}$ system, $v^2 \approx 0.2$, and is comparable with $\alpha_s(m_c)(\approx 0.3)$, therefore relativistic corrections can not be neglected. The case of S-wave have been studied in [5]. In this paper we will consider the relativistic corrections for P-wave decay rates and give complete expressions, which are accurate to next-to-leading order in both α_s and v^2.

In NRQCD, the effect of annihilation can be taken into account by adding 4-fermion operators to NRQCD Lagrangian:

$$\delta\mathcal{L}_{4-fermion} = \sum_n \frac{f_n(\alpha_s)}{m^{d_n-4}}\mathcal{O}_n, \tag{1}$$

where the sum is over all possible local 4-fermion operators $\mathcal{O}_n$ that annihilate and create a $Q\bar{Q}$ pair, and d_n is the scaling dimension of $\mathcal{O}_n$. The short distance coefficients $f_n(\alpha_s)$ can be computed by matching perturbative amplitudes for $Q\bar{Q}$ scattering in NRQCD with the corresponding amplitudes in full QCD. The annihilation rate of a quarkonium state H to light hadrons (LH) can be written as

$$\Gamma(H \to LH) = 2Im < H|\delta\mathcal{L}_{4-fermion}|H > \tag{2}$$

At a given order in v^2, the number of matrix elements can be reduced to a finite number by using velocity scaling rules for the matrix elements [1]. These scaling rules consist of that for the operators and for the probabilities of the Fock states that give the leading contributions to the matrix elements.

To next-to-leading order in v^2, the decay width of four P-wave quarkonium states can be written as

$$\begin{aligned}
\Gamma(\chi_0 \to LH) &= 2Imf_1({}^3P_0)\frac{<\chi_0|\mathcal{O}_1({}^3P_0)|\chi_0>}{m^4} + 2Img_1({}^3P_0)\frac{<\chi_0|\mathcal{P}_1({}^3P_0)|\chi_0>}{m^6} \\
&+ 2Imf_8({}^3S_1)\frac{<\chi_0|\mathcal{O}_8({}^3S_1)|\chi_0>}{m^2} + 2Img_8({}^3S_1)\frac{<\chi_0|\mathcal{P}_8({}^3S_1)|\chi_0>}{m^4} \qquad (3)\\
\Gamma(\chi_1 \to LH) &= 2Imf_1({}^3P_1)\frac{<\chi_1|\mathcal{O}_1({}^3P_1)|\chi_1>}{m^4} \\
&+ 2Imf_8({}^3S_1)\frac{<\chi_1|\mathcal{O}_8({}^3S_1)|\chi_1>}{m^2} + 2Img_8({}^3S_1)\frac{<\chi_1|\mathcal{P}_8({}^3S_1)|\chi_1>}{m^4} \qquad (4)\\
\Gamma(\chi_2 \to LH) &= 2Imf_1({}^3P_2)\frac{<\chi_2|\mathcal{O}_1({}^3P_2)|\chi_2>}{m^4} + 2Img_1({}^3P_2)\frac{<\chi_2|\mathcal{P}_1({}^3P_2)|\chi_2>}{m^6} \\
&+ 2Imf_8({}^3S_1)\frac{<\chi_2|\mathcal{O}_8({}^3S_1)|\chi_2>}{m^2} + 2Img_8({}^3S_1)\frac{<\chi_2|\mathcal{P}_8({}^3S_1)|\chi_2>}{m^4} \qquad (5)\\
\Gamma(h \to LH) &= 2Imf_1({}^1P_1)\frac{<h|\mathcal{O}_1({}^1P_1)|h>}{m^4} \\
&+ 2Imf_8({}^1S_0)\frac{<h|\mathcal{O}_8({}^1S_0)|h>}{m^2} + 2Img_8({}^1S_0)\frac{<h|\mathcal{P}_8({}^1S_0)|h>}{m^4} \qquad (6)
\end{aligned}$$

where

$$
\begin{aligned}
\mathcal{O}_1(^3P_0) &= \frac{1}{3}\psi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})\cdot\sigma\chi\chi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})\cdot\sigma\psi \\
\mathcal{O}_1(^3P_1) &= \frac{1}{2}\psi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma)\chi\cdot\chi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}}\times\sigma)\psi \\
\mathcal{O}_1(^3P_2) &= \psi^+(-\frac{i}{2}\overleftrightarrow{D})^{(i}\sigma^{j)}\chi\chi^+(-\frac{i}{2}\overleftrightarrow{D})^{(i}\sigma^{j)}\psi \\
\mathcal{O}_1(^1P_1) &= \psi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})\chi\cdot\chi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})\psi \\
\mathcal{O}_8(^1S_0) &= \psi^+T^a\chi\chi^+T^a\psi \\
\mathcal{O}_8(^3S_1) &= \psi^+T^a\sigma\chi\cdot\chi^+T^a\sigma\psi \\
\mathcal{P}_1(^3P_0) &= \frac{1}{2}[\frac{1}{3}\psi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})\cdot\sigma\chi\chi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})\cdot\sigma\psi+h.c] \\
\mathcal{P}_1(^3P_2) &= \frac{1}{2}[\psi^+(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2(-\frac{i}{2}\overleftrightarrow{D})^{(i}\sigma^{j)}\chi\chi^+(-\frac{i}{2}\overleftrightarrow{D})^{(i}\sigma^{j)}\psi+h.c] \\
\mathcal{P}_8(^1S_0) &= \frac{1}{2}[\psi^+T^a(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi\chi^+T^a\psi+h.c] \\
\mathcal{P}_8(^3S_1) &= \frac{1}{2}[\psi^+T^a\sigma(-\frac{i}{2}\overleftrightarrow{\mathbf{D}})^2\chi\cdot\chi^+T^a\sigma\psi+h.c] \qquad (7)
\end{aligned}
$$

where $\vec{D}$ is the space component of covariant derivate D^μ, ψ and χ are two component operators of quark and antiquark respectively. Coefficients are

$$
\begin{aligned}
Imf_1(^3P_0) &= (Imf_1(^3P_0))_0\{1+\frac{\alpha_s}{\pi}[(4b_0-\frac{4n_f}{27})ln\frac{\mu}{2m} \\
&+(\frac{454}{81}-\frac{\pi^2}{144})C_A+(-\frac{7}{3}+\frac{\pi^2}{4})C_F-\frac{58}{81}n_f]\} \qquad (8)\\
Imf_1(^3P_1) &= (Imf_1(^3P_0))_0\frac{\alpha_s}{\pi}[-\frac{4n_f}{27}ln\frac{\mu}{2m}+(\frac{587}{54}-\frac{317\pi^2}{288})-\frac{16n_f}{81}] \qquad (9)\\
Imf_1(^3P_2) &= (Imf_1(^3P_2))_0\{1+\frac{\alpha_s}{\pi}[(4b_0-\frac{5n_f}{9})ln\frac{\mu}{2m} \\
&+(\frac{2239}{216}-\frac{337\pi^2}{384}+\frac{5ln2}{3})C_A-4C_F-\frac{29}{27}n_f]\} \qquad (10)\\
Imf_1(^1P_1) &= \frac{(N_c^2-4)C_F\alpha_s^3}{3N_c^2}(\frac{7\pi^2-118}{48}-ln\frac{\mu}{2m}) \qquad (11)\\
Imf_8(^3S_1) &= (Imf_8(^3S_1))_0\{1+\frac{\alpha_s}{\pi}[4b_0ln\frac{\mu}{2m}-\frac{5}{9}n_f \\
&+(\frac{133}{18}+\frac{2}{3}ln2-\frac{\pi^2}{4})C_A-\frac{13}{4}C_F+\frac{5}{n_f}(-\frac{73}{4}+\frac{67}{36}\pi^2)]\} \qquad (12)\\
Imf_8(^1S_0) &= (Imf_8(^1S_0))_0\{1+\frac{\alpha_s}{\pi}[4b_0ln\frac{\mu}{2m}-\frac{8}{9}n_f \\
&+(\frac{\pi^2}{4}-5)C_F+(\frac{479}{36}-\frac{17\pi^2}{24})C_A]\} \qquad (13)\\
Img_1(^3P_0) &= -\frac{\pi C_F\alpha_s^2}{2N_c} \qquad (14)
\end{aligned}
$$

$$Img_1(^3P_2) = 0 \tag{15}$$

$$Img_8(^3S_1) = -\frac{2\pi n_f \alpha_s^2}{9} \tag{16}$$

$$Img_8(^1S_0) = -\frac{\pi(N_c^2-4)\alpha_s^2}{3N_c} \tag{17}$$

where

$$b_0 = \frac{1}{12}(11C_A - 2n_f),$$

and $C_F = \frac{N_c^2-1}{2N_c}$, $C_A = N_c$.

Comparing with the leading order results, we have added four new first order operators $\mathcal{P}_1(^3P_0)$, $\mathcal{P}_1(^3P_2)$, $\mathcal{P}_8(^1S_0)$, and $\mathcal{P}_8(^3S_1)$, which coefficients are calculated only to leading order in α_s. The coefficients of zeroth order operators have been given in [4]. Therefore we neglect higher order terms such as $\alpha_s^2\Gamma$, $v^4\Gamma$ and $\alpha_s v^2\Gamma$, and only keep those accuracy to first order in α_s or v^2. Since the first order relativistic corrections have been involved, there are no definite relations for matrix elements of zeroth order operators $\mathcal{O}_1(^3P_J)$, $\mathcal{O}_8(^3S_1)$ (J=1,2,3), $\mathcal{O}_1(^1P_1)$ and $\mathcal{O}_8(^1S_0)$. But for first order operators, due to heavy quark spin symmetry, we have

$$<\chi_0|\mathcal{P}_1(^3P_0)|\chi_0> \; = <\chi_2|\mathcal{P}_1(^3P_2)|\chi_2>, \tag{18}$$

$$<\chi_J|\mathcal{P}_8(^3S_1)|\chi_J> \; = <h|\mathcal{P}_8(^S_0)|h> . \tag{19}$$

These two matrix elements together with eight zeroth order operators' matrix elements consist of ten nonperturbative parameters. We know that there are only two parameters H_1 and H_8 [5,6] in the case of nonrelativistic limit, which can be phenomenologically determined from two experimental data. Now we have not enough experimental values to determine all these ten parameters. But their rigorous definition has been given in NRQCD, and we expect lattice calculations can provide their numerical results.

Now we give a reasonable estimate for the theoretical errors in our expression of decay rates. The three main sources of theoretical error are the neglecting terms $\alpha_s^2\Gamma$, $v^4\Gamma$, and $\alpha_s v^2\Gamma$, which can be estimated to be 9%, 4% and 6% respectively for charmonium system. Combinig the three errors, we obtain that the theoretical error is about 12% by using the standard formulas for propagating independent errors. For bottonium system, the error is much small, therefore our formulas (3), (4), (5) and (6) can give strong theoretical predictions for P-wave quarkonia decay.

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